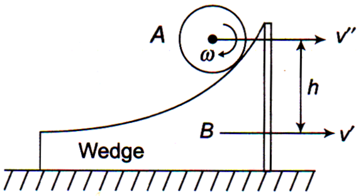
WEEKLY TEST MEDICAL PLUS -03 TEST - 12 RAJPUR
SOLUTION Date 13-10-2019

[PHYSICS]

1. At the maximum height vertical velocity of cylinder is zero, but horizontal velocity of the wedge and cylinder will be same.



In the absence of friction between the cylinder and the wedge surface, angular velocity of cylinder remains constant. From energy conservation:

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv'^2 + mgh \quad ..(i)$$

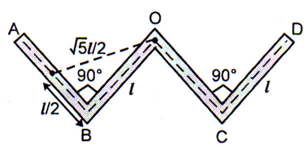
By the conservation of linear momentum

$$mv = 2mv' \quad \Rightarrow v' = v/2 \quad \ldots (ii)$$

From (i) and (ii),

$$h = v^2/4g$$

2. The given structure can be broken into 4 parts



For AB : $I = I_{CM} + m \times d^2 = \frac{m\ell^2}{12} + \frac{5m}{4}\ell^2$; $I_{AB} = \frac{4}{3}ml^2$

For BO : $I = \frac{m\ell^2}{3}$

∴ For composite frame : (by symmetry)

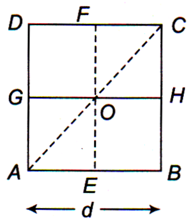
$I = 2[I_{AB} + I_{OB}] \quad = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}m\ell^2$.]

3. Let the each side of square lamina is d.

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axis,



$$I_{AC} + I_{BD} = I_0$$

$\Rightarrow \quad 2I_{AC} = I_0$ ...(i)

and $\quad I_{EF} + I_{GH} = I_0$

$\Rightarrow \quad 2I_{EF} = I_0$ ...(ii)

From Eqs. (i) and (ii), we get $I_{AC} = I_{EF}$

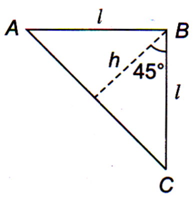
$$\therefore \quad I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left(\text{as } I_{EF} = \frac{md^2}{12}\right)$$

So, $I_{AD} = \dfrac{md^2}{3} = 4I_{EF}$

4. $h = l\cos 45° = \dfrac{l}{\sqrt{2}}$

$$I_{AC} = \frac{1}{6}Mh^2 = \frac{1}{6}M\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{Ml^2}{12}$$



5. As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

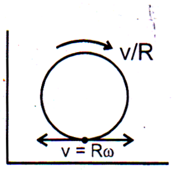
From figure

v_{net} (for lowest point) = v – Rω = v – v = 0.

and Acceleration = $\dfrac{v^2}{R} + 0 = \dfrac{v^2}{R}$

(Since linear speed is constant)

Hence (D).



6. Area between curve and displacement axis

$$= \frac{1}{2} \times (12+4) \times 10 = 80 \text{ J}$$

In this time the body acquires kinetic energy = $\frac{1}{2}mv^2$ by the law of conservation of energy

$$\frac{1}{2}mv^2 = 80 \text{ J}$$

$\Rightarrow \quad \frac{1}{2} \times 0.1 \times v^2 = 80$

$\Rightarrow \quad v^2 = 1600$

$\Rightarrow \quad v = 40$ m/s

7. We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ ...(ii)

Given, $\alpha = 3.0$ rad/s^2, $\omega_0 = 2.0$ rad/s, $t = 2$s

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or $\quad \theta = 4 + 6 = 10$ rad

8. In both the cases, the loss of gravitational potential energy and the resulting gain of 'total kinetic energy' is same.

9. $$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

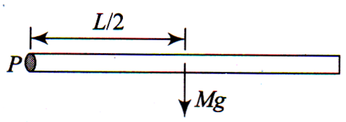
$$= \frac{300(0) + 500(40) + 400(70)}{300 + 500 + 400}$$

$$= \frac{20000 + 28000}{1200} = \frac{48000}{1200} = 40 \text{ cm}$$

10. $$\frac{1}{2} \times 3(4)^2 + \frac{1}{2} \times \frac{(3 \times R^2)}{2} \times \left(\frac{4}{R}\right)^2 = \frac{1}{2}Kx^2$$

$\Rightarrow \quad x = 0.6$ m

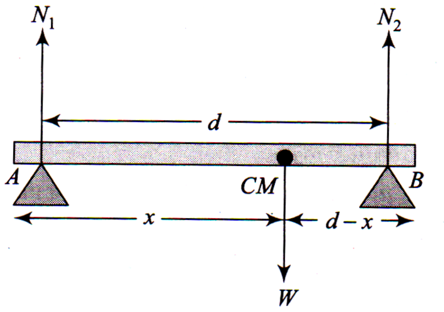
11. Taking torque about P



$$Mg\,\frac{L}{2} = \left(\frac{ML^2}{3}\right)\alpha$$

Hence $\alpha = \frac{3g}{2L}$

12 Taking torque about end A



$$\tau_B = 0$$

$$\Rightarrow \quad N_1 d = W(d-x)$$

$$\Rightarrow \quad N_1 = \frac{W(d-x)}{d}$$

13. Acceleration of sphere when it is slipping down the incline , $a_{slipping} = g \sin\theta$

Acceleration of sphere when it is rolling down

$$a_{rolling} = \frac{g \sin\theta}{1 + \frac{K^2}{r^2}} = \frac{5}{7} g \sin\theta$$

Hence required ratio $\frac{a_{rolling}}{a_{slipping}} = \frac{5}{7}$

14. $\frac{2}{3} MR_h^2 = \frac{2}{5} MR_S^2$ or $\frac{R_h^2}{R_S^2} = \frac{3}{5}$ or $\frac{R_h}{R_s} = \sqrt{\frac{3}{5}}$

15. $v = 36$ km/h = 10 m/s

By law of conservation of momentum

$$2 \times 10 = (2+3)V \Rightarrow V = 4 \text{ m/s}$$

Loss in K.E. $= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$

$= 60\,\text{J}$

From law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = (m_1+m_2)v$$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)}$$

Given,

$$m = 2\text{ kg},\ u_1 = 36 \times \frac{5}{18} = 10 \text{ m/s},$$

$$m_2 = 3\text{ kg},\ u_2 = 0$$

$$\therefore \quad v = \frac{2 \times 10 + 3 \times 0}{2+3} = 4 \text{ m/s}$$

Loss in kinetic energy is

$$\Delta K = \Delta K' - \Delta K''$$

$$= \left\{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right\} - \left\{\frac{1}{2}(m_1+m_2)v^2\right\}$$

$$= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$$

$$= 100 - 40 = 60\text{ J}$$

16. Apply law of conservation of angular momentum.

$$I_1\omega_1 = I_2\omega_2$$

In the given case $I_1 = MR^2$

and $I_2 = MR^2 = 2mR^2$

also $\omega_1 = \omega$

Then $\omega_2 = \frac{I_1}{I_1}\omega = \frac{M}{M+2m}\omega$

17. The theorem of parallel axis for moment of inertia.

$$I = I_{CM} + Mh^2$$

$$I = I_0 + M\left(\frac{L}{2}\right)^2$$

$$= I_0 + \frac{ML^2}{4}$$

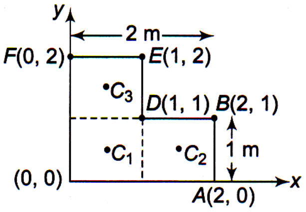
18.

By conservation of angular momentum about pivot

$L = I\omega$

$$mv\frac{d}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega \quad \Rightarrow \quad \frac{mvd}{2} = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega \quad \frac{mvd}{2} = \frac{3}{4}md^2\omega \quad \Rightarrow \frac{2}{3}\frac{v}{d} = \omega$$

19. Choosing the x and y axes as shown in the figure. The coordinates of the vertices of the L-shaped lamina is as shown in the figure.



Divide the L-shaped lamina into three squares each of side 1 m and mass 1 kg (∵ the lamina is uniform). By symmetry, the centres of mass C_1, C_2 and C_3 of the squares are their geometric centres and have coordinates $C_1\left(\frac{1}{2},\frac{1}{2}\right)$, $C_2\left(\frac{3}{2},\frac{1}{2}\right)$ and $C_3\left(\frac{1}{2},\frac{3}{2}\right)$ respectively.

The coordinates of the centre of mass of the L-shaped lamina is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{3}{2} + 1\times\frac{1}{2}}{1+1+1} = \frac{5}{6}\text{m}$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{1}{2} + 1\times\frac{3}{2}}{1+1+1} = \frac{5}{6}\text{m}$$

20. $L = mr^2\omega$, Now Given $r' = \frac{r}{2}$ and $\omega' = \omega$

$$L' = m\omega\frac{r^2}{4} = \frac{L}{4}$$

21. Speed of the rolling body at the bottom of inclined plane is

$$v=\sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

Where h is height of the inclined plane, k and R be radius of gyration and radius of the body respectively.

For solid sphere, $\frac{k^2}{R^2}=\frac{2}{5}$

$$\therefore \quad v=\sqrt{\frac{2gh}{1+\frac{2}{5}}}=\sqrt{\frac{10}{7}gh}$$

As h is same in both the cases, therefore speed will be same in both cases.

Time of descend,

$$t=\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$$

Where θ is angle of inclination?

For solid sphere, $\frac{k^2}{R^2}=\frac{2}{5}$

$$\therefore \quad t=\frac{1}{\sin\theta}\sqrt{\frac{14\,h}{5\,g}}$$

As h is same but θ is different in both the cases, hence time of descend will be different in both the cases.

22. As said, $(KE)_{\text{rot}}$ remains same.

i.e., $\frac{1}{2}I_1\omega_1^2=\frac{1}{2}I_2\omega_2^2$

$$\Rightarrow \quad \frac{1}{2I_1}(I_1\omega_1)^2=\frac{1}{2I_2}(I_2\omega_2)^2$$

$$\Rightarrow \quad \frac{L_1^2}{I_1}=\frac{L_2^2}{I_2}$$

$$\Rightarrow \quad \frac{L_1}{L_2}=\sqrt{\frac{I_1}{I_2}}$$

but $\quad I_1=I, I_2=2I$

$$\therefore \quad \frac{L_1}{L_2}=\sqrt{\frac{I}{2I}}=\frac{1}{2}$$

or $\quad L_1:L_2=1:\sqrt{2}$

23. The moment of inertia about an axis passing through centre of mass of disc and perpendicular to its plane is

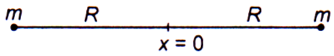
$$I_{CM} = \frac{1}{2} MR^2$$

Where M is the mass of disc and R its radius. According to theorem of parallel axis, MI of circular disc about an axis touching the disc at its diameter and normal to the disc is

$$I = I_{CM} + MR^2$$

$$= \frac{1}{2} MR^2 + MR^2 = \frac{3}{2} MR^2$$

24. For a single particle distance of centre of mass from origin is R. For more than one particles distance $\leq R$.



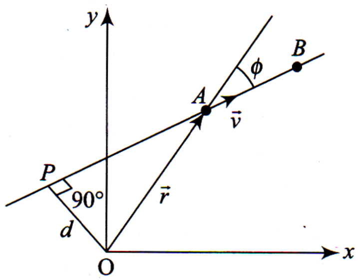
For example for two particles of equal mass, kept as shown in figure, distance = 0.

25.

(A)
$m_2 g . 1 = m_1 g . 3 \rightarrow m_2 = 3m_1$
$\rightarrow 4m_1 g . 3 = m_3 g \rightarrow m_3 = 12 m_1$
$\rightarrow 16 m_1 g . 3 = m_4 g \rightarrow m_1 = \frac{48}{48} = 1$ kg.

26. From the definition of angular momentum,

$$\vec{L} = \vec{r} \times \vec{p} = rmv \sin \phi \, (-\vec{k})$$



Therefore, the magnitude of L is

$$L = mvr \sin \phi = mvd$$

where $d = r \sin \phi$ is the distance of closest approach of the particle to the origin. As d is the same for both the cases, hence $L_A = L_B$.

27. The radius of gyration is given by

$$K = \sqrt{\frac{I}{M}}$$

For given problem $\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{I_{\text{disc}}}{I_{\text{ring}}}}$ (i)

But I_{disc} (about its axis) $= \frac{1}{2}MR^2$

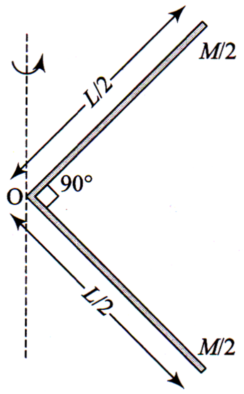
and I_{ring} (about its axis) $= MR^2$

where R is the radius of both bodies.

Therefore, Eq. (i) becomes

$$\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{\frac{1}{2}MR^2}{MR^2}} = 1:\sqrt{2}$$

28. Since rod is bent at the middle, so each part of it will have the same length $\left(\frac{L}{2}\right)$ and mass $\left(\frac{M}{2}\right)$ as shown.



29. Moment of inertia of rod of mass M and length l about its axis passing through one of its ends and perpendicular to it is

$$I = \frac{1}{3}Ml^2$$

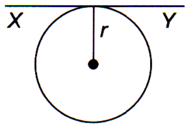
As $I = Mk^2$ where k is the radius of the gyration

$$\therefore \quad Mk^2 = \frac{1}{3}Ml^2 \text{ or } k = \frac{l}{\sqrt{3}}$$

30. Apply parallel axis theorem

$$I = I_{cm} + Mh^2 = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7ML^2}{48}$$

31. $L = 2\pi r \Rightarrow r = \dfrac{L}{2\pi}, m = \rho L$



$I_{xy} = \dfrac{3}{2} mr^2$

$= \dfrac{3}{2}\rho L\left(\dfrac{L}{2\pi}\right)^2 = \dfrac{3}{8}\dfrac{\rho L^3}{\pi^2}$

32. $I = I_0 + 6I'$

I_0 is the moment of inertia of central disc and I' is moment of inertia of rest of the each disc about specified axis.

By parallel axes theorem

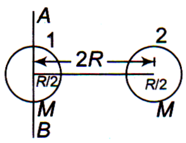
$$I = \frac{mr^2}{2} + 6(I_{cm} + md^2) = \frac{mr^2}{2} + 6\left[\frac{mr^2}{2} + m(2r)^2\right]$$

$$= \frac{mr^2}{2} + \frac{6mr^2}{2} + 24mr^2 = \frac{55mr^2}{2}$$

33. $I_{AB} = I_{1AB} + I_{2AB}$

$$= \frac{2}{5}M\left(\frac{R}{2}\right)^2 + \frac{2}{5}M\left(\frac{R}{2}\right)^2 + M(2R)^2$$

$$= \frac{21}{5}MR^2$$



34. Given $a_A = 2\alpha = 5 \text{ m/s}^2$

$\Rightarrow \quad \alpha = \dfrac{5}{2} \text{ rad/s}^2$

$\Rightarrow \quad a_B = 1 \cdot (\alpha) = 5/2 \text{ m/s}^2$

35. Here, $v_0 = 420 \text{ rpm} = 7 \text{ rps}$

$\therefore \quad \omega_0 = 2\pi v_0 = 2 \times \dfrac{22}{7} \times 7 = 44 \text{ rad s}^{-1}$

$\omega = 0, \alpha = -2 \text{ rad s}^{-2}$

$t = \dfrac{\omega - \omega_0}{\alpha} = \dfrac{-44}{-2} = 22 \text{ s}$

36. The angular displacement will be

$$\Delta\theta = \omega_{avg}\Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t$$

$$= \left(\frac{12.00 \text{ rad/s} + 4.00 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 32.0 \text{ rad}$$